SLIP EFFECTS IN OSCILLATORY FLOW OF VISCOELASTIC LIQUIDS

Ondřej WEIN

Institute of Chemical Process Fundamentals,
Czechoslovak Academy of Sciences, 165 02 Prague 6 - Suchdol

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Theory has been submitted of measurement of complex viscosity, accounting, appart from the inertia of liquid, and compliance of the instrument measuring the oscillatory shear stress, also for the effect of the slip of liquid on the wall of the measuring cell.

Modern oscillatory rheometers for testing viscoelastic materials in region of low overall shear stresses are, as far as their mechanical and electronic part is concerned, so perfect that the data on complex viscosity carry, in favourable cases, inaccuracy of only a few percent. In addition, primary data may be processed to give information about nonlinear dynamic properties[1-4]. These options of the instrument, however, can effectively be made use of provided that the processing of the primary data takes into account also secondary effects. Secondary effects, unfortunately, cannot be completely eliminated in the realization of the rheometric experiment.

Typical secondary effects represent inertia of the oscillating liquid and, further, compliance and inertia of the rheometer measuring the oscillatory shear stress in liquid. For common geometric configurations these effects have been already analyzed with relatively good results and relevant information may be found in standard handbooks[5-7].

Nevertheless, the effect of apparent slip on the wall, which may prove important[8-10] in steady-state viscometry, has not been so far investigated as a possible source of systematic deviations of measurements of complex viscosity. In the submitted paper we shall analyze slip effects assuming that even under the conditions of oscillatory flow the dynamics of the slip may be characterized by a linear relationship between the instantaneous slip velocity, $u_s(t)$, and the instantaneous shear stress, $\tau(t)$, on the slip surface, *i.e.* the wall of the instrument. Thus

$$u_s(t) = \chi\tau(t) . \qquad (1)$$

The coefficient χ is taken to be a material constant dependent both on the quality of liquid and the wall.

Let us consider first an oscillatory flow of a Newtonian liquid of viscosity μ in an h wide slot between two parallel slabs. One of the slabs is immobile while the other performs harmonic oscillations with instantaneous deviation from the mean position $l_d(t)$. Thus

$$l_d(t) = L \sin(\omega t) . \tag{2}$$

After neglecting the inertia effect in the oscillating liquid, the instantaneous shear stress may be taken equal in all points of liquid and identical with the value determined from the force reaction of both slabs confining the liquid. Since for a Newtonian liquid the shear stress is in phase with the shear rate, $\tau(t) = \mu\gamma(t)$ and, according to the assumption(1), also with the slip velocity, we may clearly write

$$h\gamma(t) = \dot{l}_d(t) - 2\chi\tau(t) = \omega L \cos(\omega t) - 2\chi\mu\gamma(t) . \tag{3}$$

The actual shear rate is thus proportional to the nominal value $\gamma_0(t)$

$$\gamma_0(t) = \omega L h^{-1} \cos(\omega t) \tag{4}$$

but it is diminished by the effect of the slip on both walls

$$\gamma(t) = \gamma_0(t)/(1 + 2\mathrm{Sl}) . \tag{5}$$

The simplex

$$\mathrm{Sl} = \chi\mu/h \tag{6}$$

characterizes thus the relative effect of the slip on one wall as the ratio of the real thickness, h, of the layer of liquid, to the thickness $\chi\mu$, corresponding to the velocity difference $\Delta u = \gamma\chi\mu$ due to the shearing deformation of liquid, the velocity difference being equal the slip velocity under the given conditions. For a liquid of constant χ and μ the relative effect of the slip, according to Eqs (5) and (6), is independent of the magnitude of the shear rate.

The measurement of slip velocities under the steady state conditions suggests that for concentrated polymer solutions the values of $\chi\mu$ may amount up to several tenths of milimeters[9,10]. For commonly used widths of the slot of oscillatory viscometers, not exceeding one millimeter, values of the simplex Sl ranging from hundredths up to tenths may be regarded as typical.

Formulation of the Problem

For an estimate of the effect of the slip on the measurement of viscoelastic response we shall consider a unidimensional model situation sketched in Fig. 1.

The tested material fills the slot of width h between a pair of parallel slabs 3, 4, located in the planes $z = 0$ and $z = h$ and performing oscillatory motion in the direction of the axis x. The driving slab 3 performs, relatively to the immobile base 7, forced harmonic oscillations with an instantaneous deviation $l_d(t)$ from the equilibrium position, expressed by Eq. (2). The measuring slab 4 is pushed into equilibrium position 0′, relatively to the immobile base 7′, by a linear elastic part 6. Its rigidity is characterized by the constant K, indicating the ratio between the tangential shear stress, τ, and the deviation, l_m, from the equilibrium position under the steady state conditions, $l_m = K\tau$. Under oscillatory motion also the inertia, I, of the mechanical part of the measuring device should be taken into consideration, characterized together with the rigidity, K, by the following equation of motion

$$K l_m(t) + I \ddot{l}_m(t) = \tau_h(t)\,, \tag{7}$$

where $\tau_h(t)$ is the course of the shearing stress on the boundary surface between the liquid and the slab of the measuring device.

The slip effect may cause that the real velocities of liquid near the wall v_0, v_h, differ from those of the walls, $\dot{l}_d$, $\dot{l}_m$, the difference being the slip velocity

$$\dot{l}_d - v_0 = \chi_d \tau_0 \tag{8a}$$

$$v_h - \dot{l}_m = \chi_m \tau_h \tag{8b}$$

with the constant slip coefficients χ_d, χ_m.

The dynamics of the oscillatory flow of liquid in the volume of the slot shall be examined within the framework of approximation of linear viscoelasticity, characterized by the relaxation modulus, $G(s)$, and by incorporating the inertia effects of

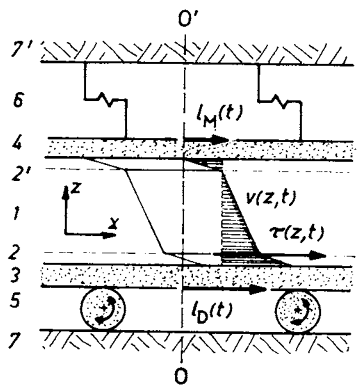

FIG. 1

Sketch of the oscillation experiment

liquid of density ϱ. Resulting equations of motion for unidimensional flows take the familiar form

$$\varrho \partial_t v(z, t) = -\partial_z \tau(z, t), \tag{9}$$

$$\tau(z, t) = \int_0^\infty G(s)\, \gamma(z, t - s)\, ds, \tag{10}$$

$$\gamma(z, t) = -\partial_z v(z, t), \tag{11}$$

and, together with the boundary conditions (*8a, b*), supplemented with the conditions of periodicity, $v(z, t + \Delta t) = v(z, t)$, $\tau(z, t + \Delta t) = \tau(z, t)$, with the period $\Delta t = 2\pi/\omega$, represent, for given $l_d(t)$ and given dynamics of the measuring device, Eq. (*7*), an unambiguously defined boundary value problem. For its solution we shall take the fields τ, v, γ and the transient course of the deflections of the measuring device, $l_m(t)$.

Representation of the Solution by Complex modules

Representation of a harmonic function of time, $f(t)$, as the real part of the product of a complex module, f^*, and complex frequency, $\exp(i\omega t)$,

$$f(t) = f_c \cos(\omega t) + f_s \sin(\omega t) = f^* \exp(i\omega t), \tag{12}$$

where

$$f^* = f_c - i f_s = |f^*| \exp(-i\phi) \tag{13}$$

is fairly commonly utilized for description of the periodic response functions in electronics[11], heat transfer[12], rheology[5-7]. As an advantage of this representation appears that the determination of the amplitude, $|f^*|$ and the phase lag, ϕ, of the harmonic function, $f(t)$, defined usually as a solution of the differential or integral equations with time variable, reduces to simple algebraic manipulation following the rules of multiplication and addition of complex numbers.

The problem under investigation is linear and the forcing function, $l_d(t)$, is harmonic. All time-variable fields, $v(z, t)$ $\tau(z, t)$, $\gamma(z, t)$, as well as the remaining time-variable parameters of the problem, *e.g.* $l_m(t)$, are harmonic functions of time. Accordingly, they may be represented by the way suggested in Eq. (*12*). The principal reason for the representation of the examined problem in terms of complex modules is the possibility of solving the problem generally, for an arbitrary linear viscoelastic material. For a harmonic course of the shearing velocity, $\gamma(t) = \gamma^* \exp(i\omega t)$, the stress response, according to the constitutive Eq. (*10*), is also harmonic, $\tau(t) = \tau^* \exp(i\omega t)$. The relationship between appropriate complex modules τ^*, γ^* may be expressed in a simple manner

$$\tau^* = \eta^* \gamma^* \tag{14}$$

in which the rheologic properties of liquid are represented by the complex viscosity $\eta^*(\omega)$, or, by the corresponding pair of real material functions $\eta^V(\omega)$, $\eta^E(\omega)$

$$\eta^*(\omega) = \int_0^\infty G(s) \exp(-i\omega s) \, ds = \eta^V(\omega) - i\eta^E(\omega) \,. \tag{15}$$

In the problems on harmonic oscillatory motion of materials, considering their inertia and linear viscoelastic behaviour, a fundamental role is played by the complex thickness of the boundary layer, δ^*

$$(\delta^*)^2 = \eta^*/(i\varrho\omega) \,. \tag{16}$$

For oscillations of bodies in infinite liquid, the real part of δ^* characterizes the distance from the body where, owing to the dissipation, substantial attenuation of the amplitude of velocity and stress occurs. The imaginary part of δ^* characterizes the wavelength and corresponding velocity of spread of the disturbance into the bulk of liquid. For oscillatory flow of given frequency, ω, in a slot h wide the character of interference of the deformation and inertia forces is determined by a single complex criterion of rheodynamic similarity[6,7] $H = h/\delta^*$. For this criterion we may write, in accord with Eq. (*16*), that

$$H^2 = i\varrho\omega h^2/\eta^* \,. \tag{17}$$

Complex modules, characterizing harmonic time course of the stress, velocity and shear rate, shall be introduced as conveniently normalized functions of complex variable Z

$$v(z, t) = \omega L V(Z) \exp(i\omega t) \,, \tag{18a}$$

$$\gamma(z, t) = \gamma_0 \Gamma(Z) \exp(i\omega t) \,, \tag{18b}$$

$$\tau(z, t) = \eta^* \gamma_0 T(Z) \exp(i\omega t) \,, \tag{18c}$$

where

$$Z = z/\delta^* = Hz/h \,, \tag{19}$$

$$\gamma_0 = \omega L/h \,. \tag{20}$$

The time variable t shall be measured from a suitable instance so as to preserve validity of the description of the motion of the driving slab as in Eq. (*2*)

$$l_d(t) = -iL \exp(i\omega t), \tag{21a}$$

$$l_m(t) = -iL_m \exp(i\omega t). \tag{21b}$$

With this assumption the parameter L (and hence also γ_0) is a real number, while L_m generally assumes complex values.

Substitution of the given definitions in the equations of motion, Eqs (*9*), (*10*) and (*11*), leads to the following set of equations

$$\Gamma(Z) = T(Z) = -HV'(Z), \tag{22}$$

$$V(Z) = -H^{-1}T'(T), \tag{23}$$

where the apostrophe indicates derivative with respect to the complex variable Z. The resulting form of the boundary conditions read

$$(K - \omega^2 I)(-iL_m) = \eta^* \gamma_0 \, T(H) \tag{24}$$

$$\omega L - \omega L V(0) = \chi_d \eta^* \gamma_0 \, T(0) \tag{25a}$$

$$\omega L V(\mathrm{H}) - \omega L_m = \chi_m \eta^* \gamma_0 \, T(H). \tag{25b}$$

The material parameters η^E, η^V are not really well suited for normalization of the boundary conditions and introduction of criteria of rheodynamic similarity. Their values may significantly vary, following a change of frequency, a fact that would considerably complicate interpretation of results for asymptotic cases $\omega \to 0$ and $\omega \to \infty$. Instead of η^E, η^V we shall therefore utilize their following combination

$$\lambda(\omega) = \eta^E(\omega)/(\omega\eta^V(\omega)), \tag{26a}$$

$$\mu(\omega) = \eta^V(\omega)(1 + \omega^2\lambda^2(\omega)). \tag{26b}$$

The parameters λ and μ vary little with frequency and for $\omega \to 0$ or $\omega \to \infty$ take on nonvanishing asymptotic values. Thus we arrive at a set of real-valued criteria

$$\mathrm{De} = \lambda\omega = \eta^E(\omega)/\eta^V(\omega) \tag{27}$$

$$\psi = \varrho\lambda^{-1}h^2/\mu \tag{28}$$

$$\mathrm{Cp} = \mu\lambda^{-1}h^{-1}(K + \omega^2 I)^{-1} \tag{29}$$

$$\mathrm{Sl}_{\mathrm{d,m}} = \chi_{\mathrm{d,m}}\mu h^{-1} \tag{30a, b}$$

and their complex analogs

$$H^2 = \mathrm{De}(\mathrm{i} - \mathrm{De})\,\psi \tag{31}$$

$$\mathrm{Cp}^* = \frac{\eta^*\omega}{\mathrm{i}h(K - \omega^2 I)} = \frac{\mathrm{Cp\,De}(\mathrm{i} + \mathrm{De})}{\mathrm{i}(1 + \mathrm{De}^2)} \tag{32}$$

$$\mathrm{Sl}^*_{\mathrm{d,h}} = \frac{\chi_{\mathrm{d,h}}\eta^*}{h} = \frac{1 - \mathrm{i\,De}}{1 + \mathrm{De}^2}\,\mathrm{Sl}_{\mathrm{d,h}}\,, \tag{33}$$

appearing in the normalized boundary value problem with the following boundary conditions

$$1 - V(0) = \mathrm{Sl}^*_{\mathrm{d}}\,T(0)\,, \tag{34a}$$

$$V(H) - L_{\mathrm{m}}/L = \mathrm{Sl}^*_{\mathrm{m}}\,T(H)\,, \tag{34b}$$

$$-L_{\mathrm{m}}/L = -\mathrm{Cp}^*\mathrm{i}\,T(H)\,. \tag{34c}$$

Formal solution takes the form of a linear superposition of exponential functions of complex variable $(H - Z)$

$$T(Z) = E^{-1}[\cosh(H - Z) + H(\mathrm{Cp}^*\mathrm{i} - \mathrm{Sl}^*_{\mathrm{m}})\sinh(H - Z)] \tag{35}$$

$$V(Z) = E^{-1}[H^{-1}\sinh(H - Z) + (\mathrm{Cp}^*\mathrm{i} + \mathrm{Sl}^*_{\mathrm{m}})\cosh(H - Z)] \tag{36}$$

$$(L_{\mathrm{m}}/L) = \mathrm{Cp}^*\mathrm{i}E^{-1} \tag{37}$$

with

$$E = [1 + H^2\mathrm{Sl}^*_{\mathrm{d}}(\mathrm{Cp}^*\mathrm{i} + \mathrm{Sl}^*_{\mathrm{m}})]\,H^{-1}\sinh H + [\mathrm{Sl}^*_{\mathrm{d}} + \mathrm{Sl}^*_{\mathrm{m}} + \mathrm{Cp}^*\mathrm{i}]\cosh H\,. \tag{38}$$

For the special case $\mathrm{Sl}^*_{\mathrm{d}} = \mathrm{Sl}^*_{\mathrm{m}} = 0$ the solution is identical with the results presented, *e.g.* in monographs[6,7] (with parameters $(H - Z) = \mathrm{i}\alpha z$, $H = \mathrm{i}\alpha h$).

RESULTS

Under ideal conditions ($H = 0$, $\mathrm{Sl}^*_{\mathrm{d,m}} = 0$, $\mathrm{Cp}^* = 0$, $E = 1$), solution of the problem under consideration becomes rather trivial. $T(Z) = 1$, $V(Z) = 1 - z/h$, and the following complex representation of the preliminary processing of the harmonic response signal

$$-\mathrm{i}L_m(K - \omega^2 I)\, h/(\omega L) = \eta_a^*(\omega) = \eta_a^V - \mathrm{i}\eta_a^E \tag{39}$$

gives, according to Eq. (*37*), directly the complex viscosity, for given frequency $\eta_a^* = \eta^*(\omega)$. Under the real conditions, when the mentioned criteria are nonvanishing, the determination of η^* necessitates solution of the complex equation (*37*), which may be written in the form $\eta^* \mathrm{E}^{-1} = \eta_a^*$, or, as set of two real transcendent equations with known right hand sides η_a^V, η_a^E and the real pair of roots η^E, η^V.

$$\begin{aligned} \eta^V c^V(\eta^V, \eta^E) &= \eta_a^V \\ \eta^E c^E(\eta^V, \eta^E) &= \eta_a^E . \end{aligned} \tag{40a, b}$$

The structure of this set follows from the definition (*32*), (*39*) and from the results (*37*) and (*38*). The correction coefficients, c^V, c^E, according to these equations, may be conveniently expressed as real functions of the arguments η^V, η^E through the components of the complex determinant $E = E_R + \mathrm{i}\, E_I$

$$\begin{aligned} c^V &= (E_R - \mathrm{De}\, E_I)/(E_R^2 + E_I^2) \\ c^E &= (E_R + \mathrm{De}^{-1}\, E_I)/(E_R^2 + E_I^2) \end{aligned} \tag{41a, b}$$

given as functions of real parameters ψ, De, Cp, $\mathrm{Sl}_{m,p}$

$$\begin{aligned} E_R &= (1 - A_1)\, B_1 - A_2 B_2 + A_3 B_3 - A_4 B_4 \\ E_I &= (1 - A_1)\, B_2 + A_2 B_1 + A_3 B_4 + A_4 B_3 , \end{aligned} \tag{42a, b}$$

$$\begin{aligned} A_1 &= \mathrm{Sl_d} \psi\, \mathrm{De}^2 (1 + \mathrm{De}^2)^{-1} (\mathrm{Cp} - \mathrm{Sl_m}) \\ A_2 &= \mathrm{Sl_d} \psi\, \mathrm{De} (1 + \mathrm{De}^2)^{-1} (\mathrm{Cp}\, \mathrm{De}^2 + \mathrm{Sl_m}) \\ A_3 &= (1 + \mathrm{De}^2)^{-1} (\mathrm{Cp}\, \mathrm{De}^2 + \mathrm{Sl_m} + \mathrm{Sl_d}) \\ A_4 &= \mathrm{De} (1 + \mathrm{De}^2)^{-1} (\mathrm{Cp} - \mathrm{Sl_m} - \mathrm{Sl_d}) \end{aligned} \tag{43a, b, c, d}$$

$$\begin{aligned} B_1 &= (\alpha \sinh \alpha \cos \beta + \beta \cosh \alpha \sin \beta)\, (\alpha^2 + \beta^2)^{-1} \\ B_2 &= (\alpha \cosh \beta \sin \beta - \beta \sinh \alpha \cos \beta)\, (\alpha^2 + \beta^2)^{-1} \\ B_3 &= \cosh \alpha \cos \beta \\ B_4 &= \sinh \alpha \sin \beta . \end{aligned} \tag{44}$$

Parameters α, β here indicate the real and the imaginary part of H,

$$H = \alpha + \mathrm{i}\beta = (\mathrm{Re}/2)^{1/2} \{[1 - \mathrm{De}/(1 + \mathrm{De}^2)^{1/2}]^{1/2} + \mathrm{i}[1 + \mathrm{De}/(1 + \mathrm{De}^2)^{1/2}]^{1/2}\} , \tag{45}$$

where

$$\mathrm{Re} = |H^2| = \psi\,\mathrm{De}\sqrt{(1 + \mathrm{De}^2)}\,. \tag{46}$$

Ignoring the effect of inertia, $\psi = 0$, and assuming that the slip properties of both boundary surfaces are equal, $\chi_m = \chi_d, = \psi$, $\mathrm{Sl}_d = \mathrm{Sl}_m = \mathrm{Sl}$, the simultaneous effect of compliance of the measuring device and slip may be expressed through the correction coefficients c^V, c^E in a substantially simpler manner

$$c^V = (1 + 2\,\mathrm{Sl})\,\Delta^{-2}\,, \quad c^E = (1 + \mathrm{Cp})\,\Delta^{-2}\,, \tag{47a, b}$$

$$\Delta^2 = \left[(1 + 2\,\mathrm{Sl} + \mathrm{Cp} + \mathrm{De}^2)^2 + \mathrm{De}^2(2\,\mathrm{Sl} - \mathrm{Cp})^2\right](1 + \mathrm{De}^2)^{-2}\,. \tag{47c}$$

From here already follow clear asymptotic estimates for the region of predominantly viscous $\mathrm{De} \rightarrow 0$

$$c^V = (1 + 2\,\mathrm{Sl})/(1 + 2\,\mathrm{Sl} + \mathrm{Cp})^2\,, \tag{48a, b}$$

$$c^E = (1 + \mathrm{Cp})/(1 + \mathrm{Cp} + 2\,\mathrm{Sl})^2\,,$$

and for the region of predominantly elastic response, $\mathrm{De} \rightarrow \infty$

$$c^V = 1 + 2\,\mathrm{Sl}\,, \tag{49a, b}$$

$$c^E = 1 + \mathrm{Cp}\,.$$

DISCUSSION

In the following discussion we shall confine ourselves to the case when both walls of the rheometer display the same properties, $\mathrm{Sl}_m = \mathrm{Sl}_d = 0$. In the set of Eqs (*30a,b*) then explicitly appear four real parameters De, ψ, Cp, Sl which are quantitative measure of the effects considered. For Maxwell's model of viscoelastic behaviour[7] the parameters μ, λ appear to be material constants. However, even for more realistic courses of the material functions $\eta^V(\omega)$, $\eta^E(\omega)$ both $\mu(\omega)$ and $\lambda(\omega)$ vary little with frequency and for $\omega \rightarrow 0$ and $\omega \rightarrow \infty$ take finite values.

Let us first examine the question of interference of the inertia and viscoelastic effects. These effects have been represented in the model by a single complex criterion, H, and correspond to a pair of real criteria ψ and De. Current meaning of the Reynolds number has Re according to Eq. (*46*).

For a common rheometric experiment one usually varies frequency of oscillations for a given liquid and configuration. To this procedure corresponds the change of De at constant ψ, Sl and (sufficiently far from the region of resonant frequency of the measuring device) also Cp.

For the case Sl = 0, Cp = 0 free of the slip and compliance effects, the dependence of c^E and c^V on frequency is shown in Fig. 2*a*, *b*. The dependence of c^V on De agrees with the common notion that for $\omega \to 0$ the effect of the inertia forces on the measured response of the rheometer becomes negligible. As a criterion fully controlling the extent of the secondary effects appears here Re. The broken line in Fig. 2*a* indicates the course for Re = 1·8 for which $c^V \approx 0{\cdot}9$.

The dependence of c^E on De is at odds with the common concept that for $\omega \to 0$ the effect of inertia may be neglected in the processing of the dynamic data. Owing to the fact that for $\omega \to 0$ both the elastic and the inertia forces are of the order $0(\omega^2)$, its ratio at ψ = const., is constant. Corresponding analytical result, according to Eq. (*41a*), for De → 0 reads

$$c^E = 1 + \mathrm{Cp} + \psi(\tfrac{1}{6} + 2\,\mathrm{Sl}) + 0(\mathrm{De})\,. \tag{50}$$

Fig. 2*b* shows also the line Re = 1·8, corresponding to an increase of c^E by about 30% for Re → 0. For values of Re above 4·5, c^V is generally negative and $c^E > 2$. This region is probably inaplicable for dynamic rheometry.

A possible effect of slip, with the inertia of liquid ignored ($\psi = 0$), on the response of the rheometer, is described by Eqs (*47a, b, c*) and corresponding asymptotic representations (*48a, b*), (*49a, b*). It is apparent that the slip effects may decrease the nonelastic component of the response (for De ≪ 1) or increase (for De ≫ 1) by a factor of (1 + 2 Sl); the elastic component may be decreased (for De ≪ 1) by a factor of $(1 + 2\,\mathrm{Sl})^2$. For certain polymer solutions (*e.g.* 2% solution of carboxymethylcellulose[8,10]) under steady state conditions the obtained values were $\mu\chi$ 0·2 mm, while for others (1% solution of polyethylene oxide[13]) these values are clearly

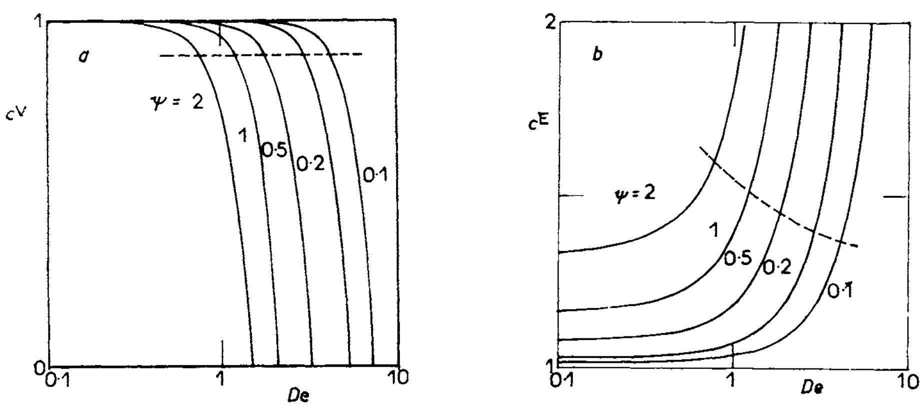

FIG. 2

The effect of sole inertia (Sl = 0, Cp = 0) on the correction factors c^V, c^E. Broken line shows the contour Re = 1·8, numbers on individual curves indicate values of ψ

below 0·01 mm. For current dimensions of the measuring cell of the oscillatory rheometer one can expect, at least for the medium concentration polymer solutions, values of the simplex Sl from the range 0·01 to 1. Available data obtained in experiments with variable h, *e.g.* ref.[14], usually display certain systematic deviations for different h. Without complete primary information, particularly the amplitude L and compliance of the measuring device, these data, however, cannot be correlated.

This circumstance becomes even more obvious when considering more realistic cases when, simultaneously with the slip, also liquid inertia plays a role. For the case of significant slip effects, $\mathrm{Sl} = 0{\cdot}5$, the quantities c^V and c^E are plotted in Figs 3*a*, *b* as functions of frequency of oscillations on several levels of inertia effects. The courses of c^V in Fig. 3*a* bring nothing new in comparison with the case $\psi = 0$. The inertia effects, considered in Fig. 2*a*, superimpose on the principal course of the dependence at $\psi = 0$ and supercritical values of $\mathrm{Re} = 1{\cdot}8$ (shown by broken line) and about ultimately a decrease of c^V to negative values. The new effect, however, becomes manifest on the courses of c^E, see Fig. 3*b*. While isolated slip or inertia effects lead to an increase of c^E with increasing frequency, their simultaneous action in the supercritical region of Re brings about finally a decrease of c^E to negative values. This turn is preceeded by an increase, which is in accord with the expected course under the influence of the inertia effects shown in Fig. 2*b*. It is thus seen that the inertia effect markedly corrects prediction of the effect of isolated slip to the value of the correction factors c^V, c^E.

It remains to add a few words about the relevance of the analyzed model situation with regard to the real rheometric experiments in axially symmetric configurations.

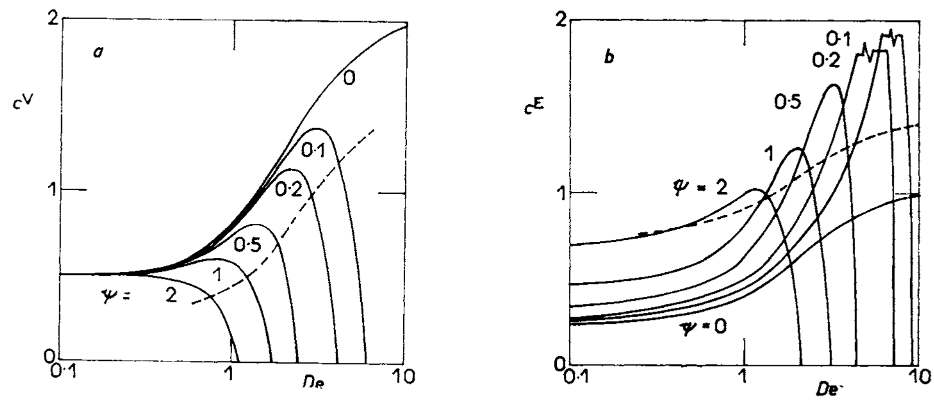

FIG. 3

Combined effect of slip and inertia for $\mathrm{Sl} = 0{\cdot}5$, $\mathrm{Cp} = 0$ on the correction factors c^V, c^E. For symbols see Fig. 2

It may be shown[6,7] that with the common neglect of the centrifugal and Weissenberg effects, *i.e.* under the assumption that oscillations do not give rise to a steady radial flow, the dynamics of the torsion oscillatory flow may be described by the same boundary value problem as the dynamics of the unidimensional flow. This conclusion remains in effect also after incorporating the slip effects in the frame work of the linear constitutive relation (*1*), which had been shown elsewhere[15].

The performed analysis showed that neglect of slip effects may lead to considerable systematic errors in the determination of complex viscosity, primarily in region of low frequency oscillations. The only way of revealing these effects is, equally as in the classic viscometry, simultaneous measurements on two configurations with different width of the slot. It has been also shown that for $\omega \to 0$ the effects of inertia cannot be generally neglected in the determination of complex viscosity.

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